

Factorising quadratic equations

When a quadratic expression is in the form $ax^2 + bx + c = 0$ find the two numbers that add to give b and multiply to give c .

$$x^2 + 7x + 10 = (x + 5)(x + 2)$$

(because 5 and 2 add to give 7 and multiply to give 10)

$$x^2 + 2x - 8 = (x + 4)(x - 2)$$

(because +4 and -2 add to give +2 and multiply to give -8)

Linear simultaneous equations

1. Balance the coefficients of one of the variables.
2. Eliminate this variable by adding or subtracting the equations (Same Sign Subtract, Different Sign Add)
3. Solve the linear equation you get using the other variable.
4. Substitute the value you found back into one of the previous equations.
5. Solve the equation you get.
6. Check that the two values you get satisfy both of the original equations.

$$5x + 2y = 9$$

$$10x + 3y = 16$$

Multiply the first equation by 2.

$$10x + 4y = 18$$

$$10x + 3y = 16$$

Same Sign Subtract (+10x on both)

$$y = 2$$

Substitute $y = 2$ in to equation.

$$5x + 2 \times 2 = 9$$

$$5x + 4 = 9$$

$$5x = 5$$

$$x = 1$$

Solution: $x = 1, y = 2$ or (1,2)



Year 10 higher topic 9 Equations and inequalities

What careers would use these skills?

Engineers, architects, economists, dieticians, paramedics, air traffic controllers, computer analysts, carpenters

Quadratic formula

$$ax^2 + bx + c = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute the values in for a , b and c , get one answer with a $+$ and then one with a $-$.

Difference of two squares

An expression in the form $a^2 - b^2$ can be factorised to give $(a + b)(a - b)$

Eg.

$$x^2 - 25 = (x + 5)(x - 5)$$

$$16x^2 - 81 = (4x + 9)(4x - 9)$$

Inequalities on a number line

Open circles are used for numbers that are less than or greater than.

Closed circles are used for numbers that are less than or equal or greater than or equal.

Integers from inequalities

State the integers that satisfy $2 < x \leq 6$ answer: 3,4,5,6

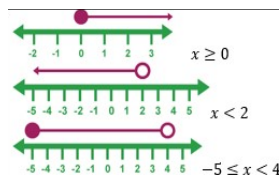
Solving linear inequalities

Treat the inequality like an equals symbol and solve it like an equations. (Remember to write the inequality symbol in your method and answer instead of an =)

Eg. Solve $2x + 5 > 11$

(subtract 5 from both sides) $2x > 6$

(divide both sides by 2) $x > 3$



Complete the square

$x^2 + bx + c$ can be written in the form

$$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

Eg. $x^2 + 6x + 7$

The completed square is

$$(x+3)^2 - 3^2 + 7$$

Which simplifies to $(x+3)^2 - 2$

Solving quadratics by completing the square

Solve $x^2 + 6x + 7 = 0$

$$(x+3)^2 - 2 = 0$$

$$(x+3)^2 = 2$$

$$x + 3 = \sqrt{2} \quad \text{and} \quad x + 3 = -\sqrt{2}$$

$$x = -3 + \sqrt{2} \quad \text{and} \quad x = -3 - \sqrt{2}$$

Wording simultaneous equations

Two numbers have a sum of 162 and they have a difference of 52. What are these two numbers?

Set up two equations and solve:

Eg. $a + b = 162$ and $a - b = 52$

($a = 107$ and $b = 55$)

Quadratic and linear simultaneous equations

Method 1: If both equations are in the same form (eg. Both $y = \dots$):

1. Set the equations **equal to each other**.
2. **Rearrange** to make the equation **equal to zero**.
3. **Solve** the quadratic equation.
4. **Substitute** the values back in to one of the equations.

Method 2: If the equations are not in the same form:

1. **Rearrange** the linear equation into the form $y = \dots$ or $x = \dots$
2. **Substitute** in to the quadratic equation.
3. **Rearrange** to make the equation **equal to zero**.
4. **Solve** the quadratic equation.
5. **Substitute** the values back in to one of the equations.

Example 1

Solve $y = x^2 - 2x - 5$ and $y = x - 1$

$$x^2 - 2x - 5 = x - 1$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = 4 \text{ and } x = -1$$

$$\text{so } y = 4 - 1 = 3 \text{ and}$$

$$y = -1 - 1 = -2$$

Answers: (4,3) and (-1,-2)

Example 2

Solve $x^2 + y^2 = 5$ and $x + y = 3$

$$x = 3 - y$$

$$(3 - y)^2 + y^2 = 5$$

$$9 - 6y + y^2 + y^2 = 5$$

$$2y^2 - 6y + 4 = 0$$

$$y^2 - 3y + 2 = 0$$

$$(y - 1)(y - 2) = 0$$

$$y = 1 \text{ and } y = 2$$

$$\text{so } x = 3 - 1 = 2 \text{ and } x = 3 - 2 = 1$$

Answers: (2,1) and (1,2)